Supplementary Materials

Three algorithms (creeping random search, simulated annealing, and pseudo-derivative) used in projection pursuit guided tour optimisation are:

Algorithm 1: Creeping random search (CRS)

input : $f(.)$, α_1, $l_{\max}$, cooling
output: $\mathbf{A}_l$
Generate random start $\mathbf{A}_1$ and set $\mathbf{A}_{\text{cur}} \coloneqq \mathbf{A}_1$, $I_{\text{cur}} = f(\mathbf{A}_{\text{cur}})$, $j = 1$;
repeat
  Set $l = 1$;
  repeat
    Generate $\mathbf{A}_l = (1 - \alpha_j)\mathbf{A}_{\text{cur}} + \alpha_j \mathbf{A}_{\text{rand}}$ and orthogonalise $\mathbf{A}_l$;
    Compute $I_l = f(\mathbf{A}_l)$;
    Update $l = l + 1$;
  until $l > l_{\max}$ *or* $I_l > I_{cur}$;
  Update $\alpha_{j+1} = \alpha_j * \text{cooling}$;
  Construct the geodesic interpolation between $\mathbf{A}_{\text{cur}}$ and $\mathbf{A}_l$;
  Update $\mathbf{A}_{\text{cur}} = \mathbf{A}_l$ and $j = j + 1$;
until $\mathbf{A}_l$ *is too close to* $\mathbf{A}_{cur}$ *in terms of geodesic distance*;

Algorithm 2: Simulated annealing (SA)

repeat
  Generate $\mathbf{A}_l = (1 - \alpha_j)\mathbf{A}_{\text{cur}} + \alpha_j \mathbf{A}_{\text{rand}}$ and orthogonalise $\mathbf{A}_l$;
  Compute $I_l = f(\mathbf{A}_l)$, $T(l) = \frac{T_0}{\log(l+1)}$ and $P = \min\left\{\exp\left[-\frac{I_{\text{cur}} - I_l}{T(l)}\right], 1\right\}$;
  Draw U from a uniform distribution: $U \sim \text{Unif}(0, 1)$;
  Update $l = l + 1$;
until $l > l_{\max}$ *or* $I_l > I_{cur}$ *or* $P > U$;

Algorithm 3: Pseudo-derivative (PD)

repeat
  Generate n random directions $\mathbf{A}_{\text{rand}}$;
  Compute $2n$ candidate bases deviate from $\mathbf{A}_{\text{cur}}$ by an angle of δ while ensuring orthogonality;
  Compute the corresponding index value for each candidate bases;
  Determine the search direction as from $\mathbf{A}_{\text{cur}}$ to the candidate bases with the largest index value;
  Determine the step size via optimising the index value on the search direction over a 90 degree window;
  Find the optimum $\mathbf{A}_{**}$ and compute $I_{**} = f(\mathbf{A}_{**})$, $p_{\text{diff}} = (I_{**} - I_{\text{cur}})/I_{**}$;
  Update $l = l + 1$;
until $l > l_{\max}$ *or* $p_{diff} > 0.001$;
